Aharonov–Bohm scattering of neutral atoms with induced electric dipole moments

Jürgen Audretsch[†,1] **and Vladimir D. Skarzhinsky**[†,‡,2]

[†]Fakultät für Physik der Universität Konstanz, Postfach M 673,
D-78457 Konstanz, Germany

[‡]P. N. Lebedev Physical Institute, Leninsky prospect 53, Moscow 117924, Russia

Abstract

We investigate the scattering of neutral polarizable atoms from an electrically charged wire placed in a homogeneous magnetic field. The atoms carry an induced electric dipole. The reflecting wire is discussed. We calculate the scattering amplitude and cross section for the practically more important case that atoms are totally absorbed at the surface of the wire. If the magnetic field is present, there is a dominating Aharonov–Bohm peak in the forward direction followed by decreasing oscillations for larger angles. An experimental realization of this modulated Aharonov–Bohm scattering should be possible.

PACS numbers: 03.65.Bz, 03.75.Be, 03.65.Nk

Aharonov and Bohm have pointed out in their famous paper [1] that electrons may interact quantum mechanically with the vector potential $\vec{A}$ in such a way that they acquire phase shifts when passing through regions where no classical electromagnetic forces are acting on them. The Aharonov–Bohm (AB) phase is a particular case of a geometrical phase [2]. The effect has been demonstrated experimentally [3]. For consequences in the domain of quantum electrodynamics see [4]. Aharonov and Casher [5] showed that the interaction of the magnetic moment of a quantum particle moving in the electric field of line charge leads to an effect of the AB type. A survey of interferometric observations of geometrical and topological phases is given in [6].

Reversing the Aharonov–Casher situation, Wilkens [7] showed that a corresponding quantum phase is to be expected for a particle with an electric dipole moment moving in the radial field of a not very realistic straight line of Dirac magnetic monopoles. A simple experimental configuration much more practical to test the quantum phase of electric dipoles moving in a combined magnetic and electric fields, was proposed by Wei, Han and Wei [8]: The electric field of a charged wire or of a cylindrical electrode polarizes a neutral atom. An uniform magnetic field is applied parallel to the wire. Moving in these fields atoms acquire a quantum phase. Below we will study the quantum scattering of these atoms in detail.

The experimental set up is the following: a neutral atom (no permanent electric dipole moment) with rest mass M_0 and electric polarizability α is moving under the combined influence of electric and magnetic fields. The electric field goes back to a charged straight wire lying along the z-axes of a cylindrical coordinate system (ρ, φ, z). $\vec{E} = E\vec{e}_\rho$ with $E = \kappa/2\pi\rho$. Superimposed is a uniform magnetic field $\vec{B} = B\vec{e}_z$ pointing in the z-direction.

[1]*e-mail: Juergen.Audretsch@uni-konstanz.de*
[2]*e-mail: vdskarzh@sgi.lpi.msk.su*

The moving atoms will be polarized by the electric field in their rest frame which is equal to $\vec{E} + (1/c)\vec{v} \times \vec{B}$ with $\vec{v}$ being the velocity of the atom. The resulting Lagrangian is

$$L = \frac{1}{2}Mv^2 + \alpha\,[\vec{B} \times \vec{E}]\frac{\vec{v}}{c} + \frac{1}{2}\alpha E^2 - U_0(\rho)\,. \tag{1}$$

We have omitted higher order of v^2/c^2 and added a potential $U_0(\rho)$ to allow for an interaction between atoms and the material of the wire. The mass $M = M_0 + \alpha B^2/c^2$ contains in addition to the rest mass M_0 a very small magnetic mass.

The Lagrangian (1) is equivalent to the Lagrangian of a charged particle with mass M and charge e moving in the effective AB vector potential $e\vec{A}(\rho) = (\hbar c\beta/\rho)\vec{e}_\varphi$ and in the electric scalar potential $U_E(\rho) = -\alpha\kappa^2/(8\pi^2\rho^2)$. The magnetic field parameter β is thereby $\beta =: \alpha\kappa B/(2\pi\hbar c)$ so that in our case the vector potential $\vec{A}$ is proportional to the magnetic field. The rotation of this effective potential vanishes so that it yields no force on a classical particle but results in a topological quantum phase (AB phase) for the quantum wave function. In this sense it may be said that we are discussing among other effects the AB scattering of an induced electric dipole.

In any case quantum scattering will not result in the pure AB scattering because the electric field of the wire, or correspondingly the influence of the scalar electric potential $U_E(\rho)$ may be essential. The influence of $U_E(\rho)$ for the case of vanishing vector potential $\vec{A}$, (no magnetic field present), has been studied by Denschlag and Schmiedmayer [9] when investigating the scattering of neutral polarizable atoms from a charged wire. They have shown that the attractive potential $U_E(\rho)$ will make atoms fall onto the wire unless the potential $U_0(\rho)$ prevents this or their angular momenta are sufficiently large to pass by the wire and go to infinity. It is the aim of this paper to work out analytically the contributions of the different physical effects to the resulting scattering and to provide expressions which can easily be evaluated numerically.

We have to find a solution of the stationary Schrödinger equation corresponding to the Lagrangian (1) (we omit the trivial z-dependence of the wave function)

$$\left\{\frac{\hbar^2}{2M}\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}\left(-i\frac{\partial}{\partial\varphi} - \beta\right)^2\right] + \frac{\alpha\kappa^2}{8\pi^2\rho^2} - U_0(\rho) + \mathcal{E}\right\}\psi(\rho,\varphi) = 0 \tag{2}$$

which satisfies the scattering boundary condition at infinity

$$\psi(\vec{p};\rho,\varphi) \sim \frac{1}{2\pi}\left[e^{ip\rho\cos(\varphi-\varphi_p)} + f(\varphi-\varphi_p)\,\frac{e^{ip\rho}}{\sqrt{\rho}}\right]\,. \tag{3}$$

The angle φ_p fixes the direction of the incoming plane wave. Below we will restrict to $\varphi_p = 0$.

The scattering wave function can be obtained by the partial decomposition

$$\psi(\rho,\varphi) = \frac{1}{\sqrt{2\pi}}\sum_{m=-\infty}^{\infty} c_m R_m(\rho)\,e^{im\varphi} \tag{4}$$

where the radial functions $R_m(\rho)$ satisfy the Bessel equation

$$R_m''(\rho) + \frac{1}{\rho}\,R_m'(\rho) - \frac{\nu^2}{\rho^2}\,R_m(\rho) - u_0(\rho)\,R_m(\rho) + p^2\,R_m(\rho) = 0\,. \tag{5}$$

Here we have introduced

$$\nu^2 := (m-\beta)^2 - \frac{\alpha\kappa^2}{(2\pi\hbar)^2}(M_0 + \alpha B^2/c^2) = m^2 - 2m\beta - \gamma^2, \tag{6}$$

the wire parameter γ

$$\gamma^2 := \frac{\alpha\kappa^2}{(2\pi\hbar)^2}M_0\,, \tag{7}$$

2

as well as
$$\hbar^2 u_0(\rho) := 2MU_0(\rho) \quad \text{and} \quad \hbar^2 p^2 := 2(M_0 + \alpha B^2/c^2)\mathcal{E}. \tag{8}$$

The scattering amplitude $f(\varphi)$ can be expressed in terms of phase shifts δ_m which are defined by the asymptotic form of the radial functions

$$R_m(\rho) \sim \sqrt{\frac{2}{\pi p \rho}} \cos\left(p\rho - \frac{\pi m}{2} - \frac{\pi}{4} + \delta_m\right). \tag{9}$$

Then the scattering amplitude reads

$$f(\varphi) = \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} f_m \, e^{im\varphi}, \quad f_m = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{p}} \, (S_m - 1), \quad S_m = e^{2i\delta_m}. \tag{10}$$

The parameter ν^2 can take negative values for any values of the parameters β and γ. This happens when m is inside a closed interval $[-m_-, m_+]$

$$-m_- \leq m \leq m_+ \quad \text{where} \quad m_\pm = [\sqrt{\beta^2 + \gamma^2} \pm \beta]. \tag{11}$$

$[a]$ denotes the integer part of a. For these negative values of ν^2 the attractive potential $U(\rho)$ predominates over the repulsive centrifugal potential and atoms can fall onto the wire acquiring an infinite negative energy if it is a string with vanishing radial extension at $\rho = 0$[10].

There are two ways to overcome this difficulty. One of them is to add a repulsive hard core potential to the external fields

$$U_0(\rho) = \begin{cases} \infty & \text{if } \rho < \rho_0 \\ 0 & \text{if } \rho > \rho_0 \end{cases} \tag{12}$$

representing a totally reflecting surface of the wire at $\rho = \rho_0$. The second way is physically more realistic and reflects the actual experimental situation: it is assumed that all polarized atoms which can not escape but finally fall onto the wire are totally absorbed by the wire. They do not take part in the scattering process furthermore. We turn now to a discussion of these two cases.

1. The totally reflecting wire

In the presence of a totally reflecting hard core potential (12) the wave function is equal to zero at $\rho = \rho_0$ and the solution of Eq.(5) has the form

$$R_m(\rho) = \frac{1}{\sqrt{2\pi}} \left[J_\nu(p\rho) - \mu_m(a) H_\nu^{(1)}(p\rho) \right], \quad \mu_m(a) = \frac{J_\nu(a)}{H_\nu^{(1)}(a)}, \quad a =: p\rho_0 \tag{13}$$

where $J_\nu(p\rho)$ and $H_\nu^{(1)}(p\rho)$ are the Bessel and the Hankel functions, correspondingly. It is easy to show that the coefficients c_m of the decomposition (4) which leads to the scattering wave function are equal to

$$c_m = e^{i\pi m - i\frac{\pi}{2}\nu}. \tag{14}$$

Then the partial scattering amplitudes read

$$f_m = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{p}} \left[e^{i\pi(m-\nu)} (1 - 2\mu_m(a)) - 1 \right]. \tag{15}$$

The hard core parameter $\mu_m(a)$ describes the additional scattering caused by the screening potential U_0. With (15) the problem can essentially be solved.

We add that we obtain for the phase shifts δ_m

$$S_m =: e^{2i\delta_m} = -e^{i\pi(m-\nu)} \frac{H_\nu^{(2)}(a)}{H_\nu^{(1)}(a)}. \tag{16}$$

Using the properties of the Hankel functions one can show that $|S_m| = 1$ at positive as well as at negative values of ν^2, and that the value of S_m is independent of the sign of the imaginary part of ν in the latter case. This means that the screening potential U_0 does indeed prevent atoms from penetrating the wire so that the scattering becomes elastic in the presence of this potential.

The scattering amplitude and the cross section depend on the parameter $a = p\rho_0$. Therefore their behavior is quite different for low and for high energies. The low energy scattering is more sensitive to the AB effect since at high energies large values of m dominate the scattering amplitude for a given angle, and one can disregard the AB parameter β in partial scattering amplitudes. For details we have to distinguish the cases of positive or negative values of ν^2.

For low energies, $(a = p\rho_0 << 1)$ and $\nu^2 \geq 0$ the phase function is asymptotically equal to

$$S_m \approx e^{i\pi(m-\nu)}, \quad \delta_m = \frac{\pi(m - \nu)}{2} \quad \text{at} \quad m \geq m_+, \ m \leq m_-, \tag{17}$$

and the hard core parameter

$$\mu_m(a) = \frac{1}{2}\left(1 - e^{-i\pi(m-\nu)} S_m\right) \approx 0 \tag{18}$$

goes to zero. These partial waves do not feel the totally reflecting potential. For $\nu^2 < 0$ the phase function can be written in the form [11]

$$S_m = e^{i\pi m}\frac{I_\nu^*(a)}{I_\nu(a)}, \quad \text{at} \quad m_- < m < m_+ \tag{19}$$

where

$$I_\nu(a) = \int_{-\infty}^{\infty} dt\, e^{i(a\cosh t + |\nu|t)} = 2\int_0^{\infty} dt\, e^{ia\cosh t}\, \cos|\nu|t. \tag{20}$$

In this case the phase shifts

$$\delta_m = \frac{\pi m}{2} + \arctan\left[\tan\left(|\nu|\ln\frac{a}{2}\right)\tan\frac{\pi|\nu|}{2}\right] \tag{21}$$

oscillate very quickly at $a \to 0$ since the low energy limit $p \to 0$ is equivalent to the case of very thin wire $\rho_0 \to 0$ for which classical atoms will spiral into the center. This leads to the situation where a small uncertainty of incoming atomic beam energy will destroy the diffraction picture.

For high energies the method of partial waves becomes ineffective and we need to use the WKB approximation to calculate the scattering cross section. In the classical limit the hard core potential $U_0(\rho)$ and the scalar potential $U(\rho)$ cause the scattering. The presence of the vector potential $A(\rho)$ gives no effect.

It seems to be not very likely realistic anyway to find an experimental realization for the scattering of atoms with an induced dipole moment by a totally reflecting electrically charged wire. Moreover our considerations show that the possibility to observe the AB effect will depend very strongly on the experimental conditions. We therefore turn now to the more promising situation of a totally absorbing wire [12].

2. The totally absorbing wire

As pointed out above we assume that atoms which fall on the wire are totally absorbed. They do not contribute to the stream of outgoing particles. These inelastic collisions will happen for partial waves with m out of the interval $[-m_-, m_+]$ for which $\nu^2 < 0$. As usual total absorption can be taken into account by using vanishing phase function $S_m = 0$ for these inelastic channels.

$$S_m = 0 \quad \text{for} \quad -m_- \leq m \leq m_+. \tag{22}$$

4

The phase functions and phase shifts for the remaining modes with elastic scattering are obtained from the radial functions

$$R_m(\rho) = \frac{1}{\sqrt{2\pi}} J_\nu(p\rho) \tag{23}$$

and are equal to

$$S_m = e^{i\pi(m-\nu)}, \quad \delta_m = \frac{\pi}{2}(m-\nu) \quad \text{for} \quad m \geq m_+ + 1, \quad m \leq -m_- - 1. \tag{24}$$

We note that $m_+ \geq m_-$.

The elastic scattering amplitude $f(\varphi)$ allows us to calculate the differential cross section for the elastic scattering

$$\frac{d\sigma^{\text{el}}(\varphi)}{d\varphi} = |f(\varphi)|^2 \tag{25}$$

Taking into account the phase function (22) and (24) from (10) we have

$$f(\varphi) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} \left[\Sigma_1(\beta, \varphi) + \Sigma_2(\beta, \varphi) + \Sigma_3(\beta, \varphi)\right] \tag{26}$$

whereby

$$\Sigma_1(\beta, \varphi) \quad := \quad \sum_{m=m_++1}^{\infty} \left(e^{i\pi(m-\nu)} - 1\right) e^{im\varphi},$$

$$\Sigma_2(\beta, \varphi) \quad := \quad \sum_{m=m_-+1}^{\infty} \left(e^{i\pi(m-\tilde\nu)} - 1\right) e^{-im\varphi}, \tag{27}$$

$$\Sigma_3(\beta, \varphi) \quad := \quad -\sum_{m=0}^{m_+} e^{im\varphi} - \sum_{m=0}^{m_-} e^{-im\varphi} + 1$$

and $\tilde\nu = \nu(-\beta)$. The expressions (27) are invariant under the substitution $\beta \to -\beta$, $\varphi \to -\varphi$. Therefore we can take $\beta > 0$. We note that the scattering amplitude is inverse proportional to $\sqrt{p}$.

It is impossible to evaluate $f(\varphi)$ analytically in a closed expression since the parameter ν depends nonlinearly on m. A numerical calculation is difficult as well, because the series (27) converge very slowly. However, the parameter ν reduces for large absolute values of m to $\nu \approx |m - \beta|$ so that the partial waves approach asymptotically the partial waves of the AB scattering amplitude

$$f_{AB}(\varphi) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} \sum_{-\infty}^{\infty} \left(e^{i\pi(m-|m-\beta|)} - 1\right) e^{im\varphi} = -\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} e^{i\frac{1}{2}\varphi} \frac{\sin \pi\beta}{\sin \frac{\varphi}{2}}. \tag{28}$$

It makes therefore sense to split $f(\varphi)$ in substracting from the sums Σ_1 and Σ_2 (and only from these) the terms of $f_{AB}(\varphi)$ with the same values of m to obtain more rapidly converging series. We combine these terms which are denoted by Σ_1^{AB} and Σ_2^{AB} with the unmodified Σ_3 to the total expression

$$f_{AB}^{\text{mod}}(\varphi) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} \left[\Sigma_1^{AB}(\beta, \varphi) + \Sigma_2^{AB}(\beta, \varphi) + \Sigma_3(\beta, \varphi)\right] \tag{29}$$

with

$$\Sigma_1^{AB}(\beta, \varphi) \quad = \quad \sum_{m=m_++1}^{\infty} \left(e^{i\pi(m-|m-\beta|)} - 1\right) e^{im\varphi},$$

$$\Sigma_2^{AB}(\beta, \varphi) \quad = \quad \sum_{m=m_-+1}^{\infty} \left(e^{i\pi(m-|m+\beta|)} - 1\right) e^{-im\varphi}. \tag{30}$$

$f_{AB}^{\mathrm{mod}}(\varphi)$ has a simple physical interpretation. By construction it is the modified AB amplitude which is obtained by taking into account the absorption of the respective partial waves. This splitting leads for the scattering amplitude $f(\varphi)$ to

$$f(\varphi) := f_{AB}^{\mathrm{mod}}(\varphi) + f_w(\varphi) \tag{31}$$

with

$$f_w(\varphi) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} \left[\delta\Sigma_1(\beta, \varphi) + \delta\Sigma_2(\beta, \varphi) \right] \tag{32}$$

whereby

$$
\begin{aligned}
\delta\Sigma_1(\beta, \varphi) &= \sum_{m=m_+ +1}^{\infty} \left(e^{i\pi(m-\nu)} - e^{i\pi(m-|m-\beta|)} \right) e^{im\varphi} \\
&= 2i e^{i\pi\beta} \sum_{m=m_+ +1}^{\infty} e^{i\frac{\pi}{2}\frac{\beta^2+\gamma^2}{m-\beta+\nu}} \sin\frac{\pi}{2}\frac{\beta^2+\gamma^2}{m-\beta+\nu} e^{im\varphi}, \\
\delta\Sigma_2(\beta, \varphi) &= \sum_{m=m_- +1}^{\infty} \left(e^{i\pi(m-\tilde\nu)} - e^{i\pi(m-|m+\beta|)} \right) e^{-im\varphi} \\
&= 2i e^{-i\pi\beta} \sum_{m=m_- +1}^{\infty} e^{i\frac{\pi}{2}\frac{\beta^2+\gamma^2}{m+\beta+\tilde\nu}} \sin\frac{\pi}{2}\frac{\beta^2+\gamma^2}{m+\beta+\tilde\nu} e^{-im\varphi}.
\end{aligned}
\tag{33}
$$

Details of the intermediate steps of the calculation are omitted.

The modified AB amplitude (29) can be easily reformulated:

$$f_{AB}^{\mathrm{mod}}(\varphi) = -\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} e^{i\frac{m_+-m_-}{2}\varphi} \frac{1}{\sin\frac{\varphi}{2}} \sin\left(\pi\beta + \frac{m_+ + m_- + 1}{2}\varphi \right). \tag{34}$$

It turns out to be the well known AB amplitude $f_{AB}(\varphi)$ with modified $\sin\pi\beta$ factor. We have now obtained the scattering amplitude $f(\varphi)$ as a sum of a closed analytical expression and two quickly converging series. We are therefore prepared for a numerical evaluation as well as a physical interpretation of the different contributions to $f(\varphi)$.

If there is a completely absorbing wire whose radius ρ_0 is large as compared to the wavelength of atoms $p\rho_0 >> 1$, then all atoms which are incident with an impact parameter $a = (m-\beta)/p < \rho_0$, or with angular momentum $|m-\beta| < p\rho_0$, will be absorbed by the wire. Accordingly we have to replace in this case the $m_\pm$ in (27) by the integer part of $p\rho_0$.

Eqs.(33) and (34) show that because the absorbing wire cuts out modes, all parts of the scattering amplitude have an oscillatory behaviour. Also the modified AB amplitude is modulated in this sense. In addition, $f_{AB}^{\mathrm{mod}}(\varphi)$ and $f_w(\varphi)$ both diverge for forward scattering $\varphi = 0$, although $f_w(\varphi)$ diverges slowlier than $f_{AB}^{\mathrm{mod}}(\varphi)$. It is therefore the region of small angles around the forward direction into which the atoms are scattered predominantly and where experimental tests should be located. We turn to this region now.

The most singular term in the forward direction arises from $f_{AB}^{\mathrm{mod}}(\varphi)$

$$f_{AB}^{\mathrm{mod}}(\varphi << 1) = -\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi p}} e^{i\frac{m_+-m_-}{2}\varphi} \frac{\sin\pi\beta}{\sin\frac{\varphi}{2}} \cos\frac{m_+ + m_- + 1}{2}\varphi. \tag{35}$$

The oscillating factor modulates the AB amplitude so that narrow peaks and deeps of the width $\delta\varphi = \pi/2\gamma$ arise which are located at $\varphi = n\pi/\gamma$ and $\varphi = (n+1/2)\pi/\gamma$, correspondingly.

The singularity of $f_w(\varphi)$ for $\varphi = 0$ is connected with large value of m. Neglecting small terms in Eq.(33) and using the formula 5.4.3(2) in [13] we obtain

$$
\begin{aligned}
\sum_{m=0}^{\infty} \frac{1}{m+\alpha} e^{im\varphi} &= \beta(\alpha)\, e^{i\alpha(\pi-\varphi)} \\
&+ \frac{1}{2} \int_{\varphi}^{\pi} e^{i[-\alpha\varphi + (\alpha - 1/2)t]} \frac{dt}{\sin(t/2)}
\end{aligned}
\tag{36}
$$

6

whereby

$$\beta(\alpha) = \frac{1}{2}\left[\psi\left(\frac{\alpha+1}{2}\right) - \psi\left(\frac{\alpha}{2}\right)\right]$$

and $\psi(\alpha)$ is the psi - function. This results in

$$f_w(\varphi << 1) \sim (\beta^2 + \gamma^2)\cos\pi\beta\ln\varphi\,. \tag{37}$$

If one neglects the magnetic mass $\alpha B^2/c^2 << M_0$, what will usually be justified, the scattering amplitude depends periodically on the magnetic field parameter β. This follows from the gauge invariance of the wave equation (2) in this case. We may therefore consider this parameter to be restricted to $0 \le \beta \le 1$. If one is interested to study how the influence of the magnetic field modifies the situation where only the charged wire is present, it is useful to compare the case $\beta = 0$ (no AB contribution) with the case $\beta = 1/2$ (maximal AB effect).

We may assume that the electric field parameter γ is larger than the restricted β. In this case β plays no role in $f_w(\varphi)$ for small angles and the switching on of the magnetic field results solely in the appearance of the modulated AB term f_{AB}^{mod}. In addition we have that the width of the oscillation is about $\pi/2\gamma$. If one want to resolve these oscillations one should choose a not too big γ. In any case we have for only small angles around the forward direction

$$\frac{f_{AB}^{\text{mod}}}{f_w} \sim (\varphi\ln\varphi)^{-1}$$

which is diverging for $\varphi = 0$, so that the appearance the AB term for the non-vanishing magnetic field should be observable in any case. If the angular resolution is good enough one can also observe the modulation of the AB cross section. If not, the cross section will be averaged and we obtain half of the AB result because the average of $\cos^2 x$ is $1/2$.

We turn to two realizations of the parameters of the experimental set up. One realization is discussed in [8]. The value of $\beta = \alpha\kappa B/(2\pi\hbar c) = 0.5$ can be obtained for alkali atoms with the polarizability $\alpha \sim 10 \times 10^{-40}\text{Fm}^2$ placed in an electric field $E \sim 10^7\text{V/m}$ of the wire of the radius $\sim 1\text{mm}$ (this corresponds to $\kappa \sim 10^4\text{V}$) and in a magnetic field $B \sim 5\text{T}$. In this case we have $\beta^2/\gamma^2 = \alpha B^2/M_0 c^2 \le 10^{-11}$. Obviously we can neglect β as compared to γ. In this case the distances between two peaks are extremely small so that the cross section will get the factor $1/2$. An estimation of γ of the order of 10 was given in [9]. In this case it should be possible to separate the peaks.

The figures show for two values of the wire parameter γ the differential cross section for small angles. The oscillations go back to the absorption of particles on the wire and are also present for vanishing magnetic field ($\beta = 0$, thin lines). Switching on the magnetic field ($\beta = 1/2$, bold lines) leads to a shift of the oscillations. What has not been shown is that for smaller angles φ the cross section with magnetic field is considerably larger for both values of γ. This goes back to the fact that the AB scattering dominates for very small angles.

Acknowledgments

References

[1] Y. Aharonov and D. Bohm, Phys. Rev. 119 (1959) 485.

[2] For a resource letter see: J.Anandan, J.Christian, and K.Wanelik, Am.J.Phys., 65 (1997) 180.

[3] For a survey see: M.Peshkin and A.Tonomura, The Aharonov–Bohm effect (Springer-Verlag, Berlin, 1989).

[4] J. Audretsch, U. Jasper, and V. D. Skarzhinsky, Phys. Rev. D 53 (1996) 2178, 2190; V.D.Skarzhinsky and J.Audretsch, J.Phys., A: Math.Gen. 30 (1997) 7603.

[5] Y.Aharonov and A.Casher, Phys.Rev.Lett. 53 (1984) 319.

[6] U.Sterr, K.Sengstock, W.Ertmer, F.Riehle, and J.Helmcke, Atominterferometry based on separated light fields in: Atom interferometry, ed P.R.Berman (Academic Press, San Diego, 1997).

[7] M.Wilkens, Phys.Rev.Lett. 72, (1994) 5.

[8] H.Wei, R.Han, and X.Wei, Phys.Rev.Lett. 75 (1995) 2071.

[9] J.Denschlag and J.Schmiedmayer, Europhys. Lett. 38 (6) (1997) 405.

[10] The related mathematical problems have been stressed in: C.R.Hagen, Phys.Rev.Lett. 77 (1996) 1656.

[11] I.S.Gradshteyn and I.M.Ryzhik, Table of integrals, series and products (New York, Academic Press, 1980).

[12] In a different way this problem has also been attacked by U.Leonhardt and M.Wilkens (private communication).

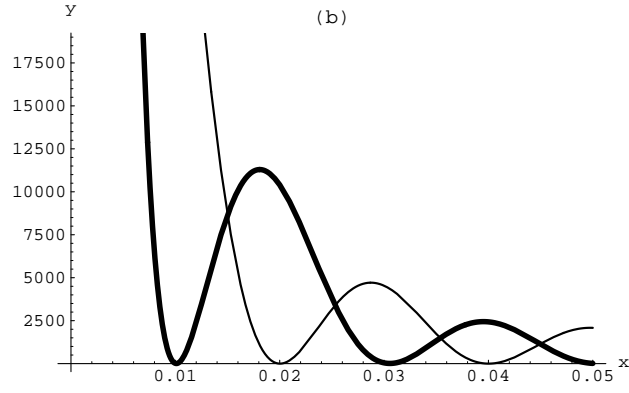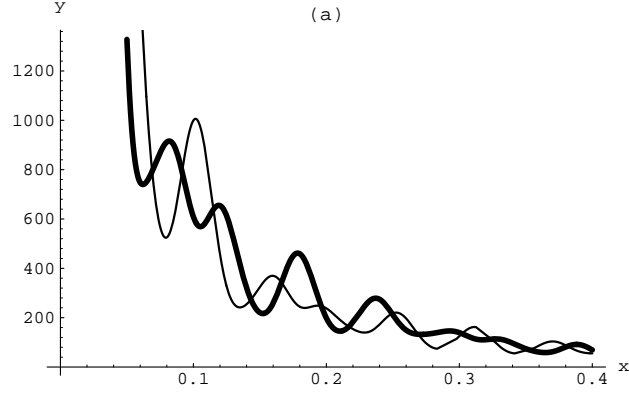[13] A.P.Prudnikov, Ju.A.Brychkov and O.I Marichev, Integrals and Series, Vol.1 (Nauka, Moscow, 1981, in Russian).

Figure 1: Differential scattering cross section ($x = \varphi, \quad y = 2\pi p |f(x)|^2$) for the case of a totally absorbing wire and vanishing magnetic field ($\beta = 0$, thin lines) or non vanishing magnetic field ($\beta = 1/2$, bold lines).The wire parameter $\gamma = 5.1$ in Figure (a) and $\gamma = 50.1$ in Figure (b).